A Logical Approach to Discrete Math

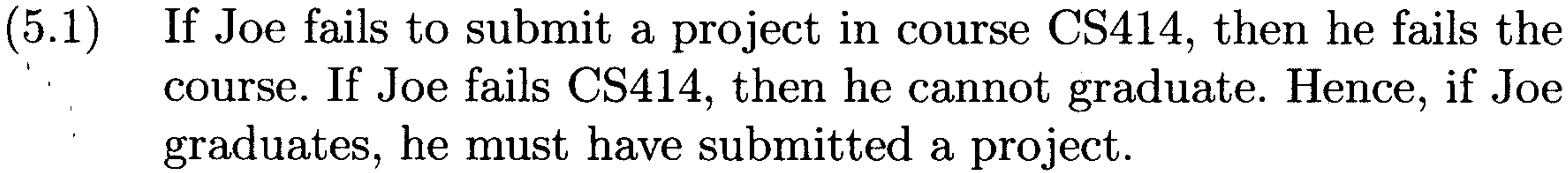
Verifying English arguments

Verifying English arguments

(5.1) If Joe fails to submit a project in course CS414, then he fails the course. If Joe fails CS414, then he cannot graduate. Hence, if Joe graduates, he must have submitted a project.

A Logical Approach to Discrete Math

Verifying English arguments



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Is (5.1) a valid argument?

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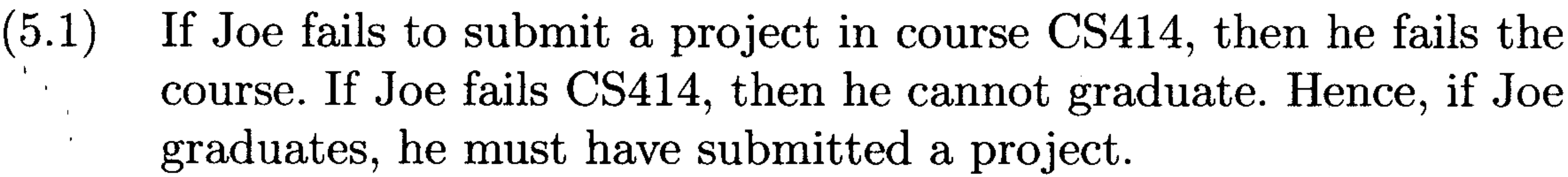

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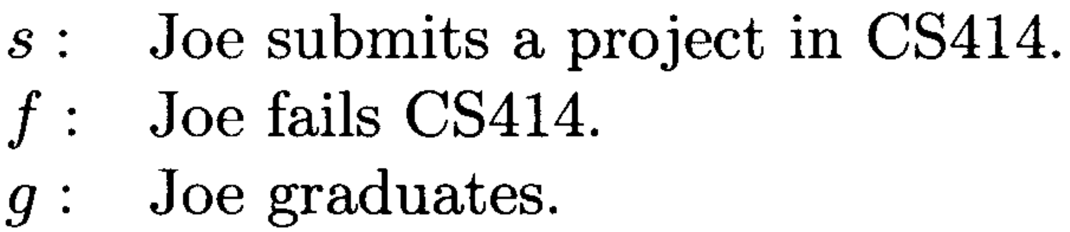

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$F0$: $\neg s \Rightarrow f$
$F1$: $f \Rightarrow \neg g$
C : $g \Rightarrow s$

Verifying English arguments

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s : Joe submits a project in CS414.
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The argument is: $F0 \wedge F1 \Rightarrow C$

$F0: \quad \neg s \Rightarrow f$
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The argument is: $F0 \wedge F1 \Rightarrow C$

It is a valid argument if the following is a theorem.

$$(\neg s \Rightarrow f) \wedge (f \Rightarrow \neg g) \Rightarrow (g \Rightarrow s)$$

A Logical Approach to Discrete Math

Prove $(\neg s \Rightarrow f) \wedge (f \Rightarrow \neg g) \Rightarrow (g \Rightarrow s)$

A Logical Approach to Discrete Math

Prove $(\neg s \Rightarrow f) \wedge (f \Rightarrow \neg g) \Rightarrow (g \Rightarrow s)$

Proof

A Logical Approach to Discrete Math

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Proof

$$(\neg s \Rightarrow f) \wedge (f \Rightarrow \neg g)$$

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Prove $(\neg s \Rightarrow f) \wedge (f \Rightarrow \neg g) \Rightarrow (g \Rightarrow s)$

Proof

$$(\neg s \Rightarrow f) \wedge (f \Rightarrow \neg g)$$

$\Rightarrow$ $\langle$(3.82a) Transitivity$\rangle$

A Logical Approach to Discrete Math

Prove $\quad (\neg s \Rightarrow f) \wedge (f \Rightarrow \neg g) \Rightarrow (g \Rightarrow s)$

Proof

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$\Rightarrow \quad \langle$(3.82a) Transitivity$\rangle$

$\quad \neg s \Rightarrow \neg g$

A Logical Approach to Discrete Math

Prove $\quad (\neg s \Rightarrow f) \wedge (f \Rightarrow \neg g) \Rightarrow (g \Rightarrow s)$

Proof

$$
\begin{array}{ll}
 & (\neg s \Rightarrow f) \wedge (f \Rightarrow \neg g) \\
\Rightarrow & \langle \text{(3.82a) Transitivity} \rangle \\
 & \neg s \Rightarrow \neg g \\
= & \langle \text{(3.61) Contrapositive} \rangle
\end{array}
$$

A Logical Approach to Discrete Math

Prove $(\neg s \Rightarrow f) \land (f \Rightarrow \neg g) \Rightarrow (g \Rightarrow s)$

Proof

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\begin{array}{ll}
 & (\neg s \Rightarrow f) \land (f \Rightarrow \neg g) \\
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 & g \Rightarrow s \quad //
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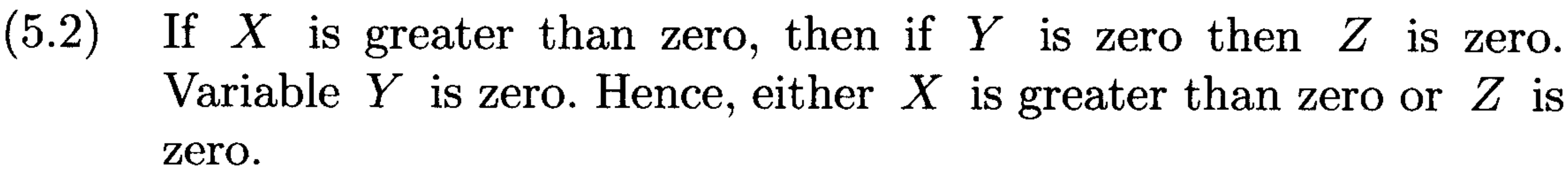
A Logical Approach to Discrete Math

(5.2) If X is greater than zero, then if Y is zero then Z is zero. Variable Y is zero. Hence, either X is greater than zero or Z is zero.

A Logical Approach to Discrete Math



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A Logical Approach to Discrete Math

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Is (5.2) a valid argument?

x : X is greater than zero.
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$F0$: $x \Rightarrow (y \Rightarrow z)$
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C :

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x : X is greater than zero.
y : Y is zero.
z : Z is zero.

$F0$: $x \Rightarrow (y \Rightarrow z)$
$F1$: y
C : $x \vee z$

A Logical Approach to Discrete Math

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C : $x \vee z$

The argument is: $F0 \wedge F1 \Rightarrow C$

A Logical Approach to Discrete Math

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It is a valid argument if the following is a theorem.

A Logical Approach to Discrete Math

(5.2) If X is greater than zero, then if Y is zero then Z is zero. Variable Y is zero. Hence, either X is greater than zero or Z is zero.

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$F1 :$ y
$C :$ $x \vee z$

The argument is: $F0 \wedge F1 \Rightarrow C$

It is a valid argument if the following is a theorem.

$$(x \Rightarrow (y \Rightarrow z)) \wedge y \ \Rightarrow \ x \vee z$$

A Logical Approach to Discrete Math

What if the argument is not valid?

Find a state for which it is false.

That state is called a counterexample.

Search for a counterexample

$$(x \Rightarrow (y \Rightarrow z)) \wedge y \;\Rightarrow\; x \vee z$$

Search for a counterexample

$$\overset{\text{T}}{\overline{(x \Rightarrow (y \Rightarrow z)) \wedge y}} \;\Rightarrow\; \overset{\text{F}}{\overline{x \vee z}}$$

Search for a counterexample

$$\overset{\text{T}}{\overline{(x \Rightarrow (y \Rightarrow z)) \land y}} \;\Rightarrow\; \overset{\text{F}}{\overline{\underset{\text{F}}{x} \lor \underset{\text{F}}{z}}}$$

Search for a counterexample

$$\overset{\text{T}}{\overline{(\underset{\text{F}}{x} \Rightarrow (y \Rightarrow \underset{\text{F}}{z})) \wedge y}} \Rightarrow \overset{\text{F}}{\overline{\underset{\text{F}}{x} \vee \underset{\text{F}}{z}}}$$

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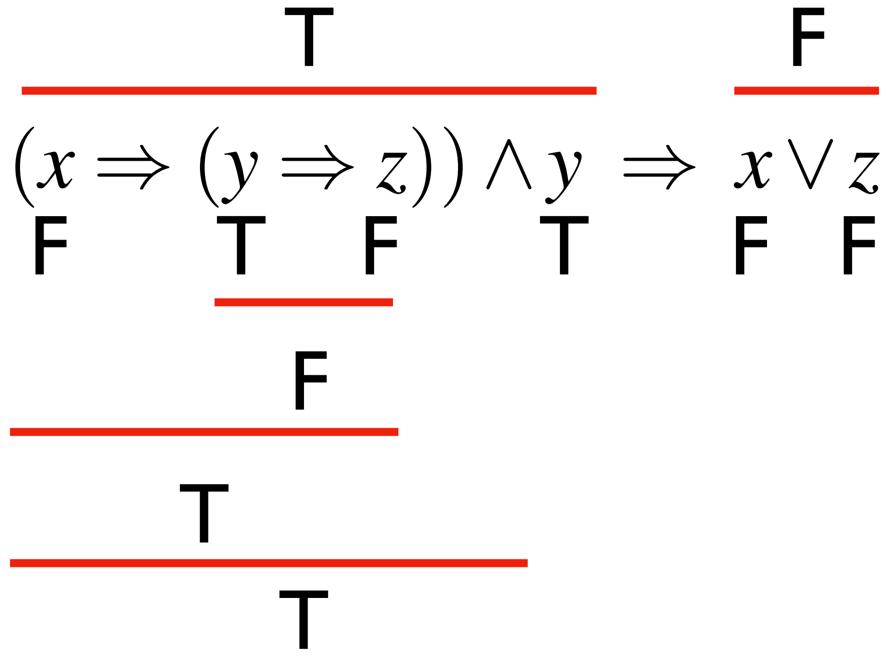
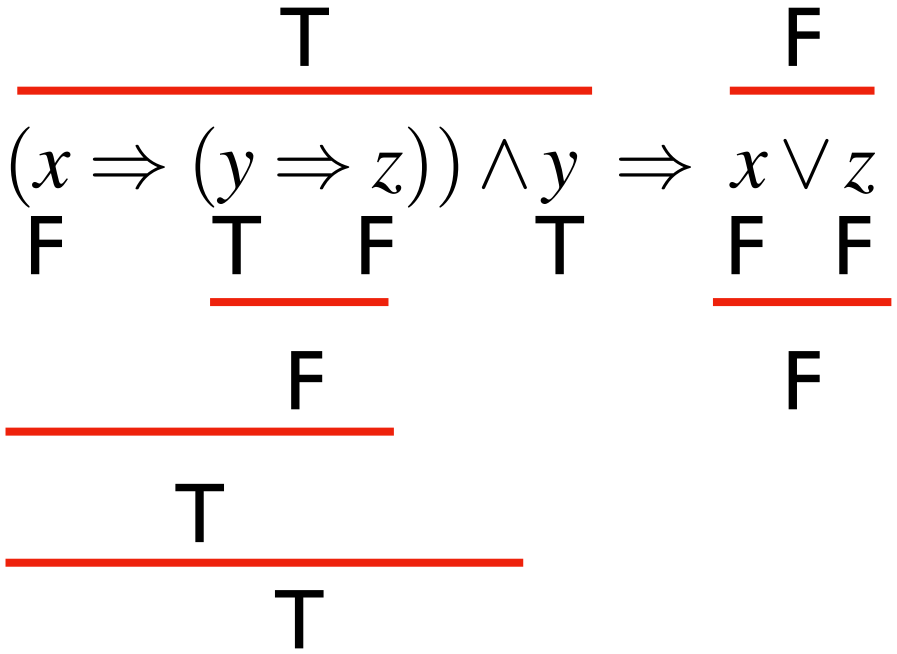
Search for a counterexample



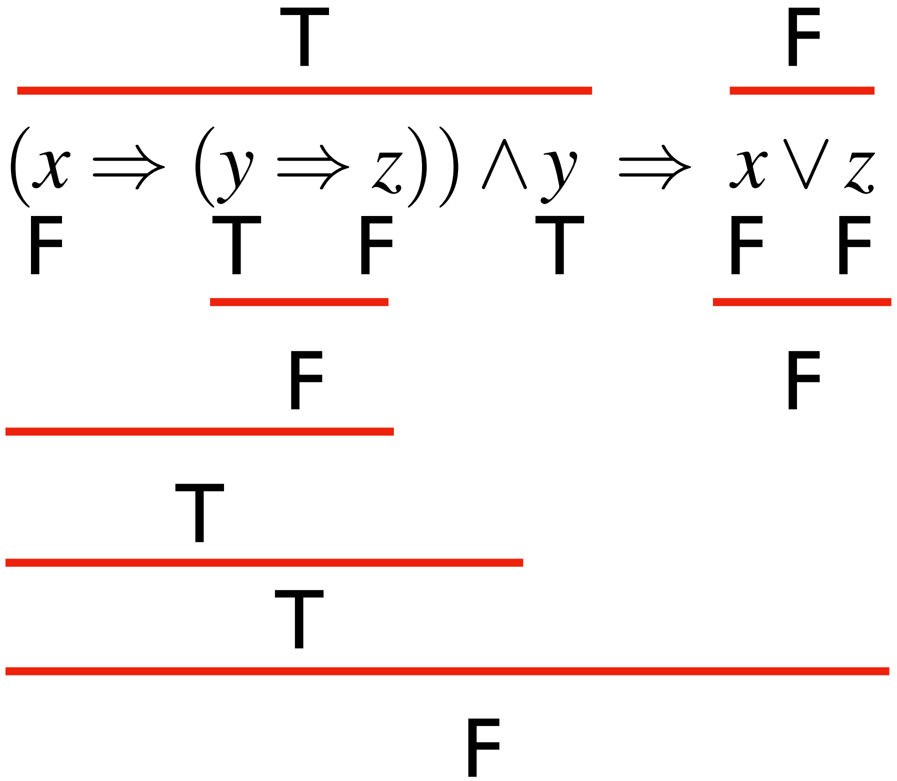
$$(x \Rightarrow (y \Rightarrow z)) \land y \Rightarrow x \lor z$$

Search for a counterexample



$$(x \Rightarrow (y \Rightarrow z)) \wedge y \Rightarrow x \vee z$$

Search for a counterexample



T F

$$(x \Rightarrow (y \Rightarrow z)) \wedge y \Rightarrow x \vee z$$

F T F T F F

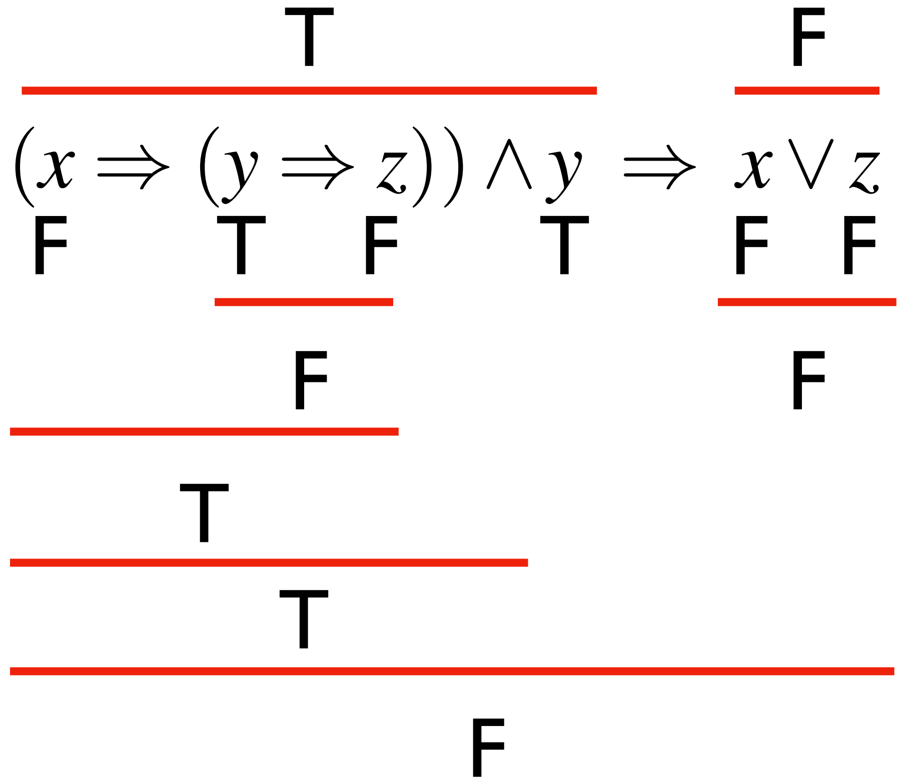
F F

T

T

F

Search for a counterexample



$$(x \Rightarrow (y \Rightarrow z)) \wedge y \quad \Rightarrow \quad x \vee z$$

Counterexample: $(x, false)$, $(y, true)$, $(z, false)$

A Logical Approach to Discrete Math

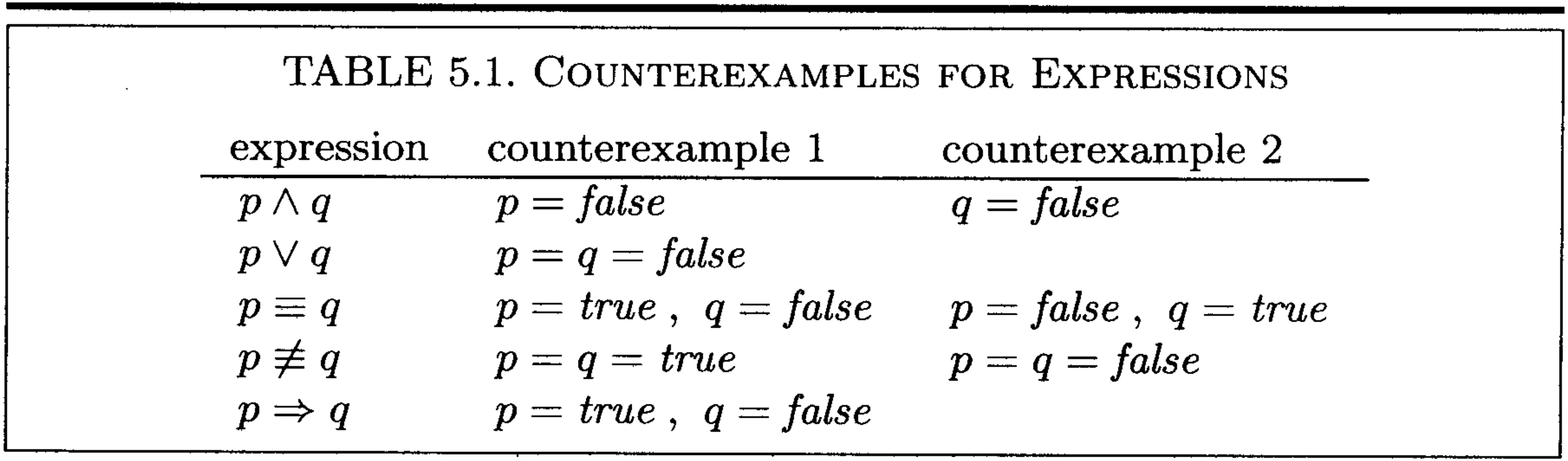
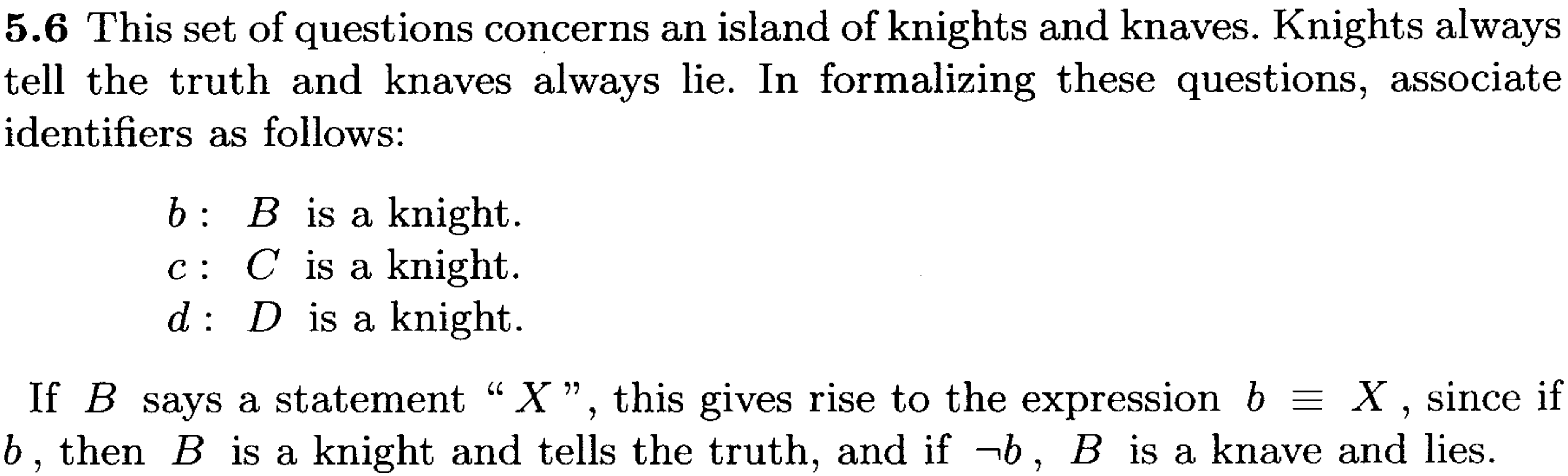


TABLE 5.1. COUNTEREXAMPLES FOR EXPRESSIONS

expression	counterexample 1	counterexample 2
$p \wedge q$	$p = \mathit{false}$	$q = \mathit{false}$
$p \vee q$	$p = q = \mathit{false}$	
$p \equiv q$	$p = \mathit{true}\ ,\ \ q = \mathit{false}$	$p = \mathit{false}\ ,\ \ q = \mathit{true}$
$p \not\equiv q$	$p = q = \mathit{true}$	$p = q = \mathit{false}$
$p \Rightarrow q$	$p = \mathit{true}\ ,\ \ q = \mathit{false}$	

A Logical Approach to Discrete Math

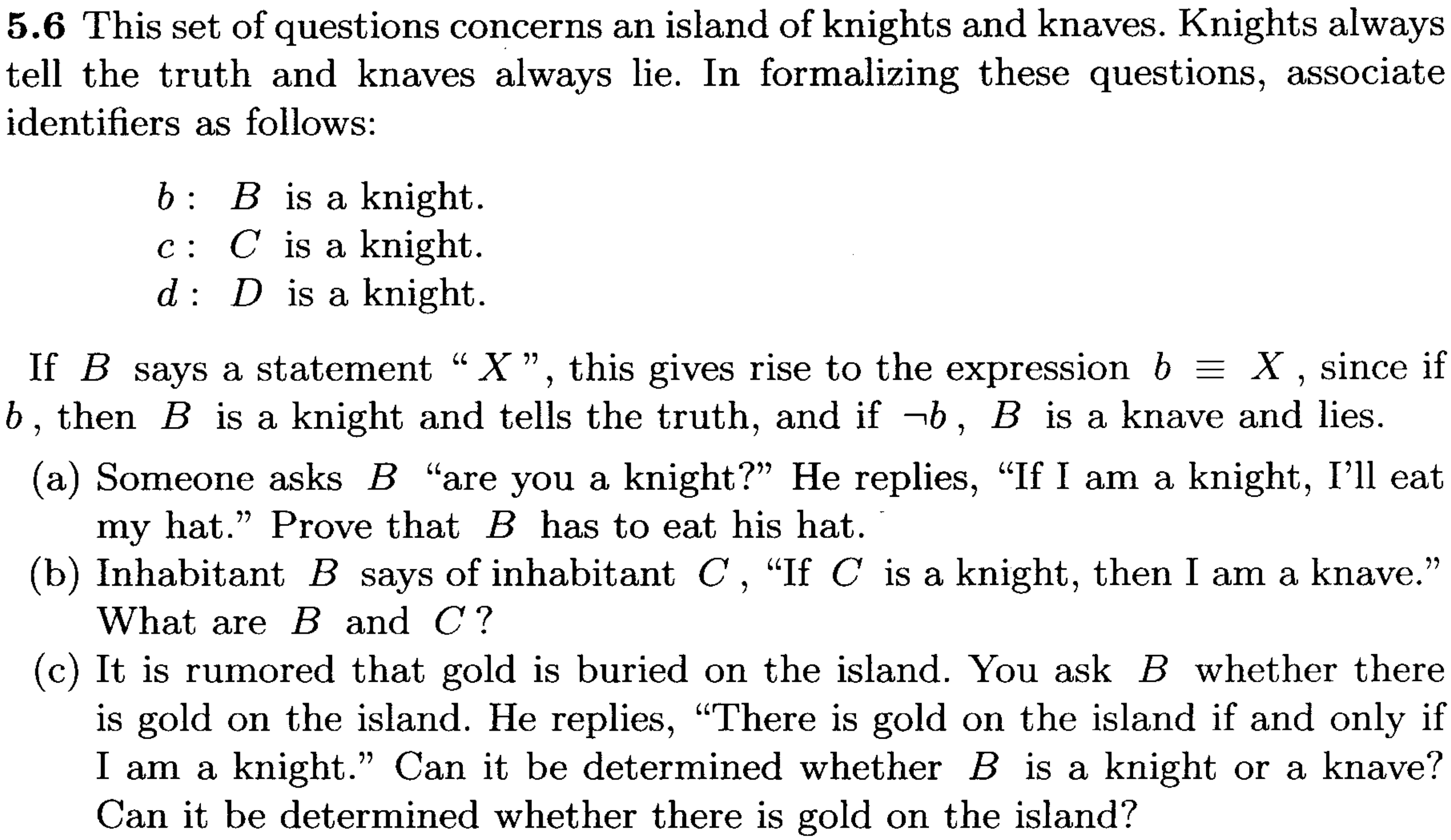


5.6 This set of questions concerns an island of knights and knaves. Knights always tell the truth and knaves always lie. In formalizing these questions, associate identifiers as follows:

b : B is a knight.
c : C is a knight.
d : D is a knight.

If B says a statement "X", this gives rise to the expression $b \equiv X$, since if b, then B is a knight and tells the truth, and if $\neg b$, B is a knave and lies.

A Logical Approach to Discrete Math



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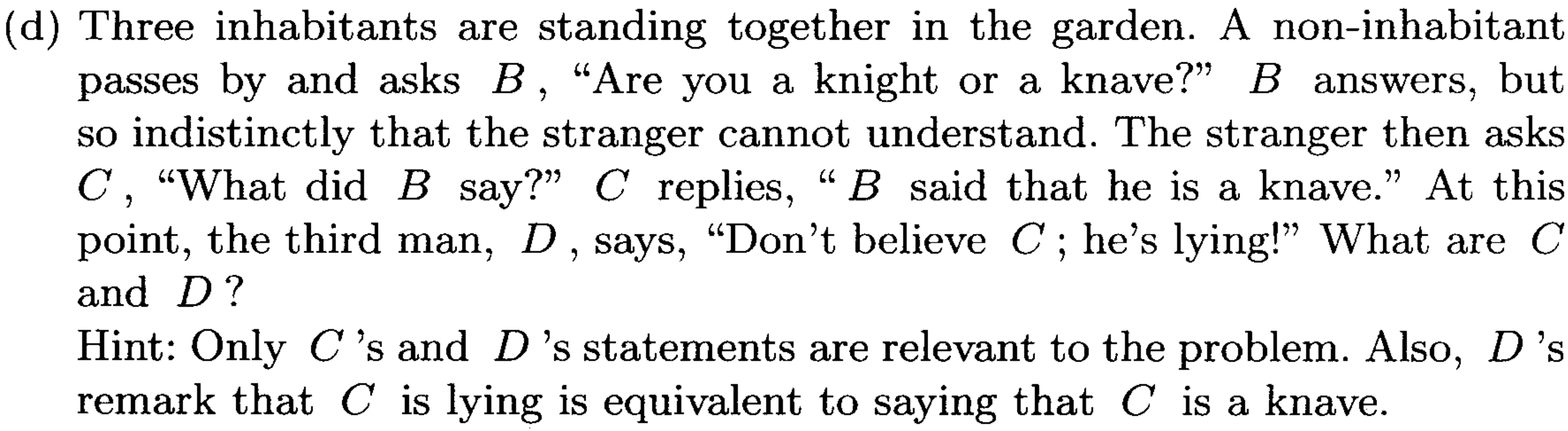
If B says a statement "X", this gives rise to the expression $b \equiv X$, since if b, then B is a knight and tells the truth, and if $\neg b$, B is a knave and lies.

(a) Someone asks B "are you a knight?" He replies, "If I am a knight, I'll eat my hat." Prove that B has to eat his hat.

(b) Inhabitant B says of inhabitant C, "If C is a knight, then I am a knave." What are B and C?

(c) It is rumored that gold is buried on the island. You ask B whether there is gold on the island. He replies, "There is gold on the island if and only if I am a knight." Can it be determined whether B is a knight or a knave? Can it be determined whether there is gold on the island?

A Logical Approach to Discrete Math



(d) Three inhabitants are standing together in the garden. A non-inhabitant passes by and asks B, “Are you a knight or a knave?” B answers, but so indistinctly that the stranger cannot understand. The stranger then asks C, “What did B say?” C replies, “B said that he is a knave.” At this point, the third man, D, says, “Don’t believe C; he’s lying!” What are C and D?

Hint: Only C’s and D’s statements are relevant to the problem. Also, D’s remark that C is lying is equivalent to saying that C is a knave.

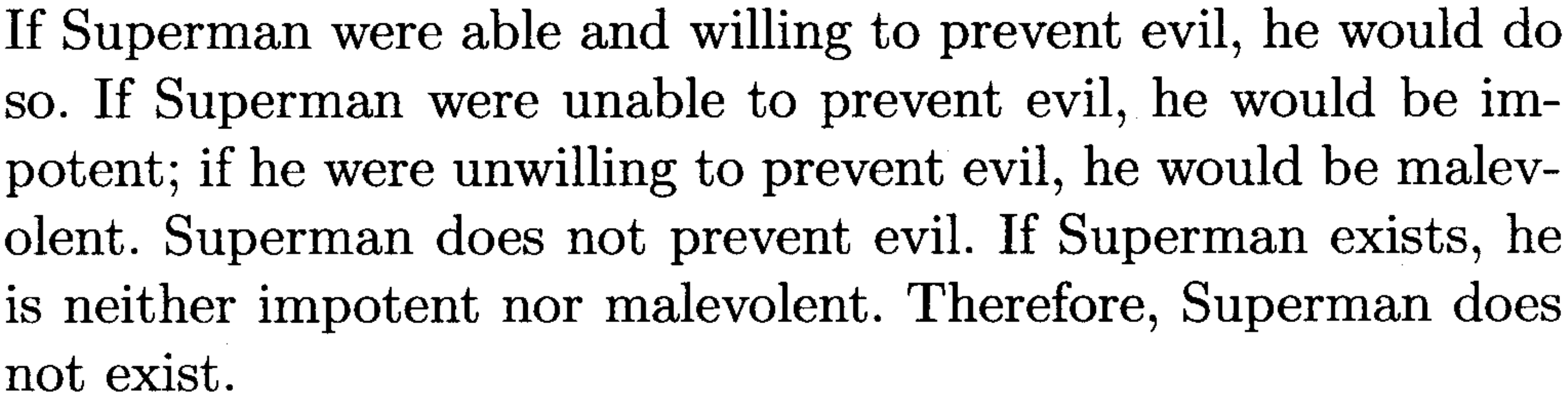
The theodicy question

If there is a powerful and merciful God, why does evil exist in the world?

Gottfried Leibniz wrote (1709)

Theodicy: Essays on the Goodness of God, the Freedom of Man and the Origin of Evil

A Logical Approach to Discrete Math



If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

A Logical Approach to Discrete Math

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$F0:$

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$$F0: \quad a \wedge w \Rightarrow p$$

A Logical Approach to Discrete Math

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$F0$: $a \wedge w \Rightarrow p$
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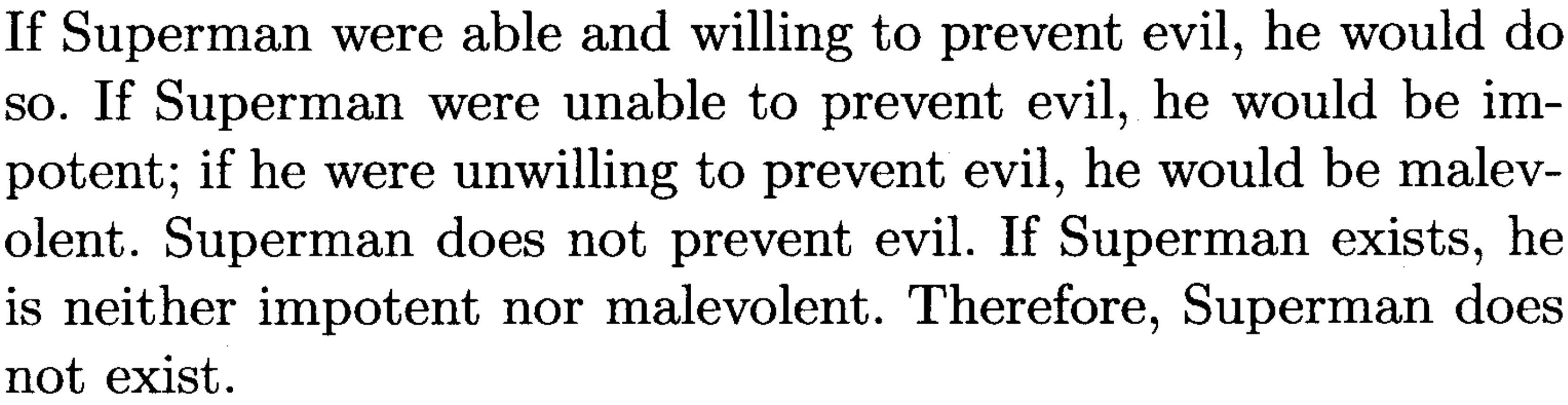
$F2: \quad \neg p$
$F3:$

A Logical Approach to Discrete Math



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$F3: \quad e \Rightarrow \neg i \wedge \neg m$

A Logical Approach to Discrete Math

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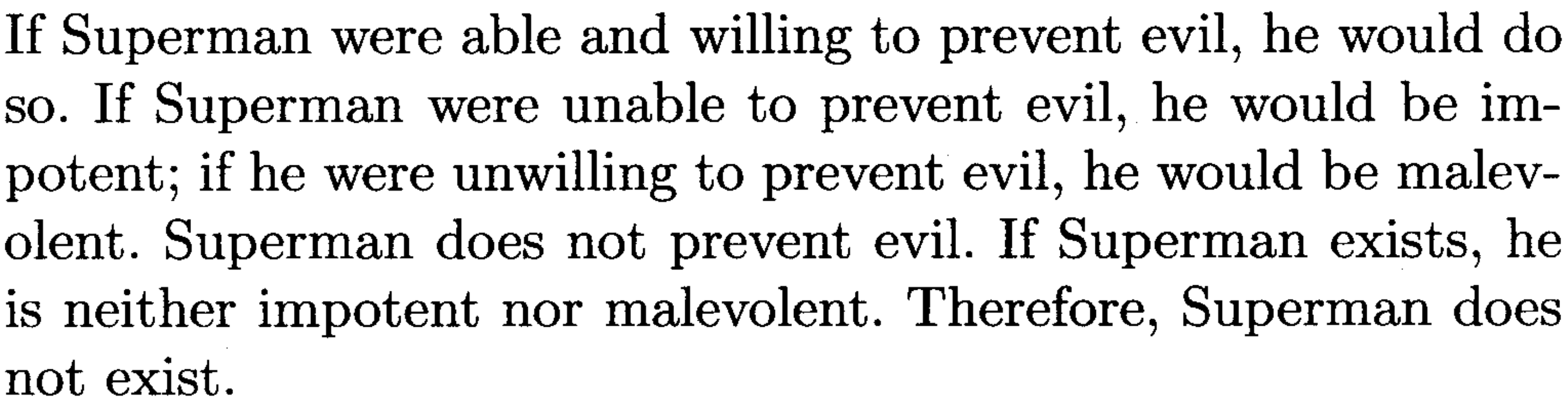
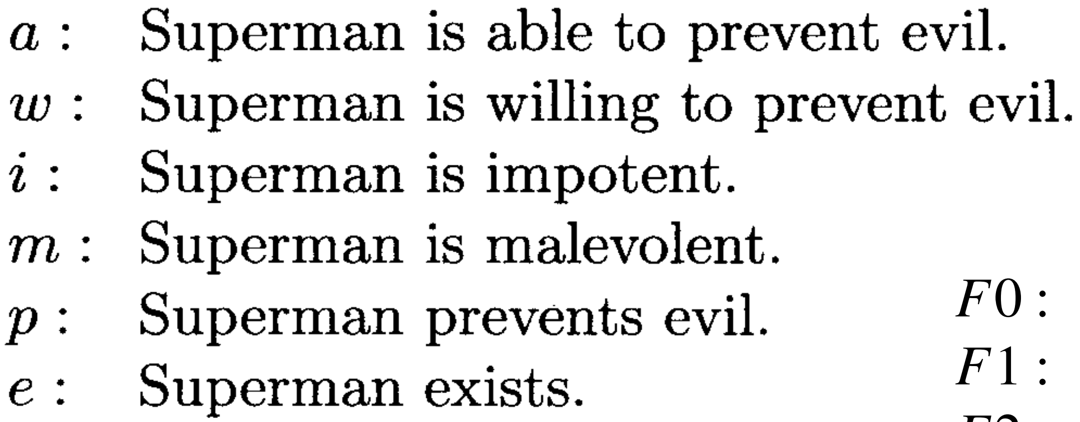
$F2$: $\neg p$
$F3$: $e \Rightarrow \neg i \wedge \neg m$
C :

A Logical Approach to Discrete Math



If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.



a : Superman is able to prevent evil.
w : Superman is willing to prevent evil.
i : Superman is impotent.
m : Superman is malevolent.
p : Superman prevents evil.
e : Superman exists.

$F0: \quad a \wedge w \Rightarrow p$
$F1: \quad (\neg a \Rightarrow i) \wedge (\neg w \Rightarrow m)$
$F2: \quad \neg p$
$F3: \quad e \Rightarrow \neg i \wedge \neg m$
$C: \quad \neg e$

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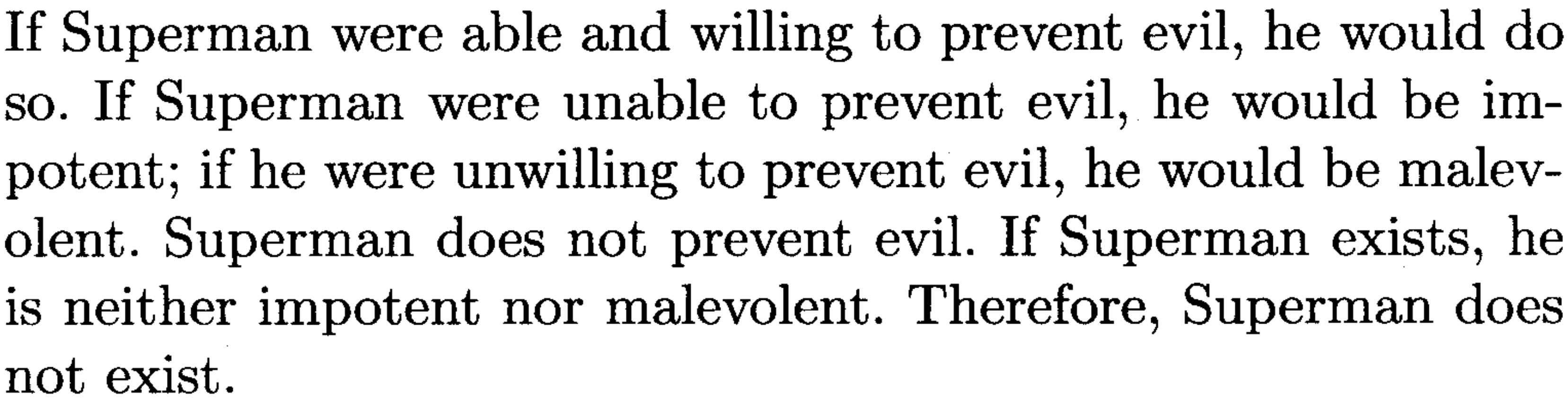
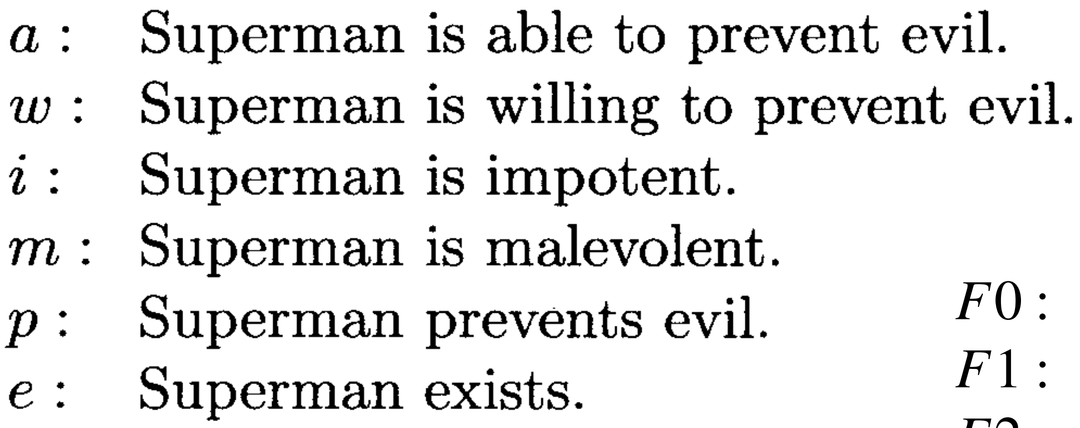


If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.



a : Superman is able to prevent evil.
w : Superman is willing to prevent evil.
i : Superman is impotent.
m : Superman is malevolent.
p : Superman prevents evil.
e : Superman exists.

$F0$: $a \wedge w \Rightarrow p$
$F1$: $(\neg a \Rightarrow i) \wedge (\neg w \Rightarrow m)$
$F2$: $\neg p$
$F3$: $e \Rightarrow \neg i \wedge \neg m$
C : $\neg e$

The argument is: $F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow C$

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$$\begin{array}{ll} F0: & a \wedge w \Rightarrow p \\ F1: & (\neg a \Rightarrow i) \wedge (\neg w \Rightarrow m) \\ F2: & \neg p \\ F3: & e \Rightarrow \neg i \wedge \neg m \\ C: & \neg e \end{array}$$

Prove $F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow C$

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$$
\begin{array}{ll}
F0: & a \wedge w \Rightarrow p \\
F1: & (\neg a \Rightarrow i) \wedge (\neg w \Rightarrow m) \\
F2: & \neg p \\
F3: & e \Rightarrow \neg i \wedge \neg m \\
C: & \neg e
\end{array}
$$

Prove $F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow C$

Proof: By (4.4) deduction $\underline{\text{and}}$ (4.7.1) truth implication.

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$$\begin{array}{ll} F0: & a \wedge w \Rightarrow p \\ F1: & (\neg a \Rightarrow i) \wedge (\neg w \Rightarrow m) \\ F2: & \neg p \\ F3: & e \Rightarrow \neg i \wedge \neg m \\ C: & \neg e \end{array}$$

Prove $F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow C$

Proof: By (4.4) deduction and (4.7.1) truth implication.

Prove $F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow (true \Rightarrow \neg e)$

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$$\begin{aligned} F0: &\quad a \wedge w \Rightarrow p \\ F1: &\quad (\neg a \Rightarrow i) \wedge (\neg w \Rightarrow m) \\ F2: &\quad \neg p \\ F3: &\quad e \Rightarrow \neg i \wedge \neg m \\ C: &\quad \neg e \end{aligned}$$

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$\neg e$

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Prove $\quad F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow C$

Proof: By (4.4) deduction <u>and</u> (4.7.1) truth implication.

Prove $\quad F0 \wedge F1 \wedge F2 \wedge F3 \;\Rightarrow\; (true \Rightarrow \neg e)$

$\quad \neg e$

$\Leftarrow \quad \langle$Assume contrapositive of conjunct $F3$,

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$$\begin{array}{ll} F0: & a \wedge w \Rightarrow p \\ F1: & (\neg a \Rightarrow i) \ \wedge \ (\neg w \Rightarrow m) \\ F2: & \neg p \\ F3: & e \Rightarrow \neg i \ \wedge \ \neg m \\ C: & \neg e \end{array}$$

Prove $F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow C$

Proof: By (4.4) deduction and (4.7.1) truth implication.

Prove $F0 \wedge F1 \wedge F2 \wedge F3 \ \Rightarrow \ (true \Rightarrow \neg e)$

$\neg e$

$\Leftarrow \quad \langle$Assume contrapositive of conjunct $F3$, $\neg(\neg i \ \wedge \ \neg m) \Rightarrow \neg e\rangle$

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$$\begin{array}{ll} F0: & a \wedge w \Rightarrow p \\ F1: & (\neg a \Rightarrow i) \ \wedge \ (\neg w \Rightarrow m) \\ F2: & \neg p \\ F3: & e \Rightarrow \neg i \ \wedge \ \neg m \\ C: & \neg e \end{array}$$

Prove $F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow C$

Proof: By (4.4) deduction <u>and</u> (4.7.1) truth implication.

Prove $F0 \wedge F1 \wedge F2 \wedge F3 \ \Rightarrow \ (true \Rightarrow \neg e)$

$$\begin{array}{ll} & \neg e \\ \Leftarrow & \langle \text{Assume contrapositive of conjunct } F3, \neg(\neg i \ \wedge \ \neg m) \Rightarrow \neg e \rangle \\ & \neg(\neg i \ \wedge \ \neg m) \end{array}$$

$$\begin{array}{ll} F0: & a \wedge w \Rightarrow p \\ F1: & (\neg a \Rightarrow i) \ \wedge \ (\neg w \Rightarrow m) \\ F2: & \neg p \\ F3: & e \Rightarrow \neg i \ \wedge \ \neg m \\ C: & \neg e \end{array}$$

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Prove $\quad F0 \wedge F1 \wedge F2 \wedge F3 \ \Rightarrow \ (true \Rightarrow \neg e)$

$$\begin{array}{ll} & \neg e \\ \Leftarrow & \langle \text{Assume contrapositive of conjunct } F3, \neg(\neg i \ \wedge \ \neg m) \Rightarrow \neg e \rangle \\ & \neg(\neg i \ \wedge \ \neg m) \\ = & \langle \text{De Morgan and double negation twice} \rangle \end{array}$$

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$$\begin{array}{ll} F0: & a \wedge w \Rightarrow p \\ F1: & (\neg a \Rightarrow i) \ \wedge \ (\neg w \Rightarrow m) \\ F2: & \neg p \\ F3: & e \Rightarrow \neg i \ \wedge \ \neg m \\ C: & \neg e \end{array}$$

Prove $F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow C$

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$$\begin{aligned}
F0 &: \quad a \wedge w \Rightarrow p \\
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F3 &: \quad e \Rightarrow \neg i \;\wedge\; \neg m \\
C &: \quad \neg e
\end{aligned}$$

Prove $\quad F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow C$

Proof: By (4.4) deduction and (4.7.1) truth implication.

Prove $\quad F0 \wedge F1 \wedge F2 \wedge F3 \;\Rightarrow\; (true \Rightarrow \neg e)$

$$\begin{aligned}
& \neg e \\
\Leftarrow \quad & \langle \text{Assume contrapositive of conjunct } F3, \neg(\neg i \;\wedge\; \neg m) \Rightarrow \neg e \rangle \\
& \neg(\neg i \;\wedge\; \neg m) \\
= \quad & \langle \text{De Morgan and double negation twice} \rangle \\
& i \;\vee\; m \\
\Leftarrow \quad & \langle \text{Assume first conjunct of } F1, \neg a \Rightarrow i \text{ and monotonicity of } \vee \rangle
\end{aligned}$$

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$$\begin{aligned} F0: &\quad a \wedge w \Rightarrow p \\ F1: &\quad (\neg a \Rightarrow i) \ \wedge \ (\neg w \Rightarrow m) \\ F2: &\quad \neg p \\ F3: &\quad e \Rightarrow \neg i \ \wedge \ \neg m \\ C: &\quad \neg e \end{aligned}$$

Prove $\quad F0 \wedge F1 \wedge F2 \wedge F3 \Rightarrow C$

Proof: By (4.4) deduction <u>and</u> (4.7.1) truth implication.

Prove $\quad F0 \wedge F1 \wedge F2 \wedge F3 \ \Rightarrow \ (true \Rightarrow \neg e)$

$$\begin{aligned} &\quad \neg e \\ \Leftarrow &\quad \langle \text{Assume contrapositive of conjunct } F3, \neg(\neg i \ \wedge \ \neg m) \Rightarrow \neg e \rangle \\ &\quad \neg(\neg i \ \wedge \ \neg m) \\ = &\quad \langle \text{De Morgan and double negation twice} \rangle \\ &\quad i \ \vee \ m \\ \Leftarrow &\quad \langle \text{Assume first conjunct of } F1, \neg a \Rightarrow i \text{ and monotonicity of } \vee \rangle \\ &\quad \neg a \ \vee \ m \end{aligned}$$

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\begin{array}{ll}
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C: & \neg e
\end{array}
$$

$\neg a \vee m$

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$$\begin{array}{ll} F0: & a \wedge w \Rightarrow p \\ F1: & (\neg a \Rightarrow i) \ \wedge \ (\neg w \Rightarrow m) \\ F2: & \neg p \\ F3: & e \Rightarrow \neg i \ \wedge \ \neg m \\ C: & \neg e \end{array}$$

$$\begin{array}{ll} & \neg a \ \vee \ m \\ \Leftarrow & \langle \text{Assume second conjunct of } F1, \neg w \Rightarrow m \text{ and monotonicity of } \vee \rangle \end{array}$$

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$$\begin{array}{ll} F0: & a \wedge w \Rightarrow p \\ F1: & (\neg a \Rightarrow i) \wedge (\neg w \Rightarrow m) \\ F2: & \neg p \\ F3: & e \Rightarrow \neg i \wedge \neg m \\ C: & \neg e \end{array}$$

$$\begin{array}{ll} & \neg a \vee m \\ \Leftarrow & \langle \text{Assume second conjunct of } F1, \neg w \Rightarrow m \text{ and monotonicity of } \vee \rangle \\ & \neg a \vee \neg w \\ = & \langle \text{De Morgan} \rangle \\ & \neg(a \wedge w) \end{array}$$

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$$\begin{array}{ll} F0: & a \wedge w \Rightarrow p \\ F1: & (\neg a \Rightarrow i) \wedge (\neg w \Rightarrow m) \\ F2: & \neg p \\ F3: & e \Rightarrow \neg i \wedge \neg m \\ C: & \neg e \end{array}$$

$\neg a \vee m$

$\Leftarrow$ ⟨Assume second conjunct of $F1$, $\neg w \Rightarrow m$ and monotonicity of $\vee$⟩

$\neg a \vee \neg w$

$=$ ⟨De Morgan⟩

$\neg(a \wedge w)$

$\Leftarrow$ ⟨Assume contrapositive of conjunct $F0$,

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$$\begin{array}{ll}
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F3: & e \Rightarrow \neg i \ \wedge \ \neg m \\
C: & \neg e
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 & \neg a \ \vee \ \neg w \\
= & \langle \text{De Morgan} \rangle \\
 & \neg(a \ \wedge \ w) \\
\Leftarrow & \langle \text{Assume contrapositive of conjunct } F0, \neg p \Rightarrow \neg(a \wedge w) \rangle
\end{array}$$

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F0: & a \wedge w \Rightarrow p \\
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$$\begin{array}{ll}
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 & \neg a \vee \neg w \\
= & \langle \text{De Morgan} \rangle \\
 & \neg (a \wedge w) \\
\Leftarrow & \langle \text{Assume contrapositive of conjunct } F0, \neg p \Rightarrow \neg (a \wedge w) \rangle \\
 & \neg p \\
= & \langle \text{Assume conjunct } F2 \rangle
\end{array}$$

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The theodicy question

Does this argument prove that God does not exist?

No, it does not.

If any of F0, F1, F2, F3 concerning the nature of God is not true, then the conclusion does not follow.