Proof for (8.22) Change of dummy.

Provided $\neg occurs(\text{`}y\text{'}, \text{`}R, P\text{'})$, and f has an inverse, $\quad (\star x \mid R : P) = (\star y \mid R[x := f.y] : P[x := f.y])$

Proof:

$(\star y \mid R[x := f.y] : P[x := f.y])$

$=$ $\langle$(8.14) One-point rule in body$\rangle$

$=$ $\langle$(8.20) Nesting, because x cannot be in $R[x := f.y]\rangle$

$=$ $\langle$(3.84a) Substitution$\rangle$

$=$ $\langle$Property of textual substitution $R[x := x] = R\rangle$

$=$ $\langle$Definition of inverse, $x = f.y \;\equiv\; y = f^{-1}.x\rangle$

$=$ $\langle$(8.20) Nesting, legal because $\neg occurs(\text{`}y\text{'}, \text{`}R\text{'})\rangle$

$=$ $\langle$(8.14) One-point rule$\rangle$

$=$ $\langle$Textual substitution because $\neg occurs(\text{`}y\text{'}, \text{`}P\text{'})\rangle$