1. Study Section 9.1.

2. Prove (9.6).
   See the hint in Exercise 9.3.

3. Prove (9.8).
   See the hint in Exercise 9.4.

4. Prove (9.9).
   See the hint in Exercise 9.5.

5. Prove (9.11) Body weakening/strengthening.
   See the hint in Exercise 9.8.

6. Prove (9.13) Instantiation: $(\forall x \mid : P) \Rightarrow P[x := E]$. Note that instantiation involves a textual substitution on the right side of an implication. The only previous axiom or theorem that relates a quantified expression with a textual substitution is the one-point rule (8.14). The problem is that the one-point rule has the restriction that x does not occur in E, while instantiation has no such restriction. So, to prove instantiation without the restriction you must first use dummy renaming (8.21), which also uses the *occurs* predicate. The proof is outlined below with the hints supplied and the proof steps for you to fill in.

   $(\forall x \mid : P)$

   $=$ $\langle$(8.21) Dummy renaming with $y, R := z, true$, and z chosen to not occur in $E\rangle$

   $=$ $\langle$(3.28) Excluded middle with $p := z = E\rangle$

   $\Rightarrow$ $\langle$(9.10) Range weakening$\rangle$

   $=$ $\langle$(8.14) One-point rule, which is legal now because z is not free in $E\rangle$

   $=$ $\langle$Property of textual substitution, because z is not free in $P\rangle$